University of Glasgow | School of Engineering

1st TERRE School

26-27 September 2016

Naples, Italy

Basic aspects of mechanical behaviour of unsaturated soils

Simon Wheeler
School of Engineering, University of Glasgow, UK

Contents

Introduction: Unsaturated Soils

Definition: soils in which the void spaces contain both liquid (typically water) and gas (typically air)

Occurrence:

- Natural soils above the water table
- Fills
- Natural gas generation

Relevance:

- Damage to structures and pavements caused by wetting and drying
- Slope instabilities and landslides
- Performance of flood embankments and earth dams
- Clay barriers for nuclear waste disposal
- Pollutant migration and containment

Pore Pressures and Suction

Pore air pressure u_a

Pore water pressure u_w

$$u_a - u_w = T(1/r_1 + 1/r_2)$$

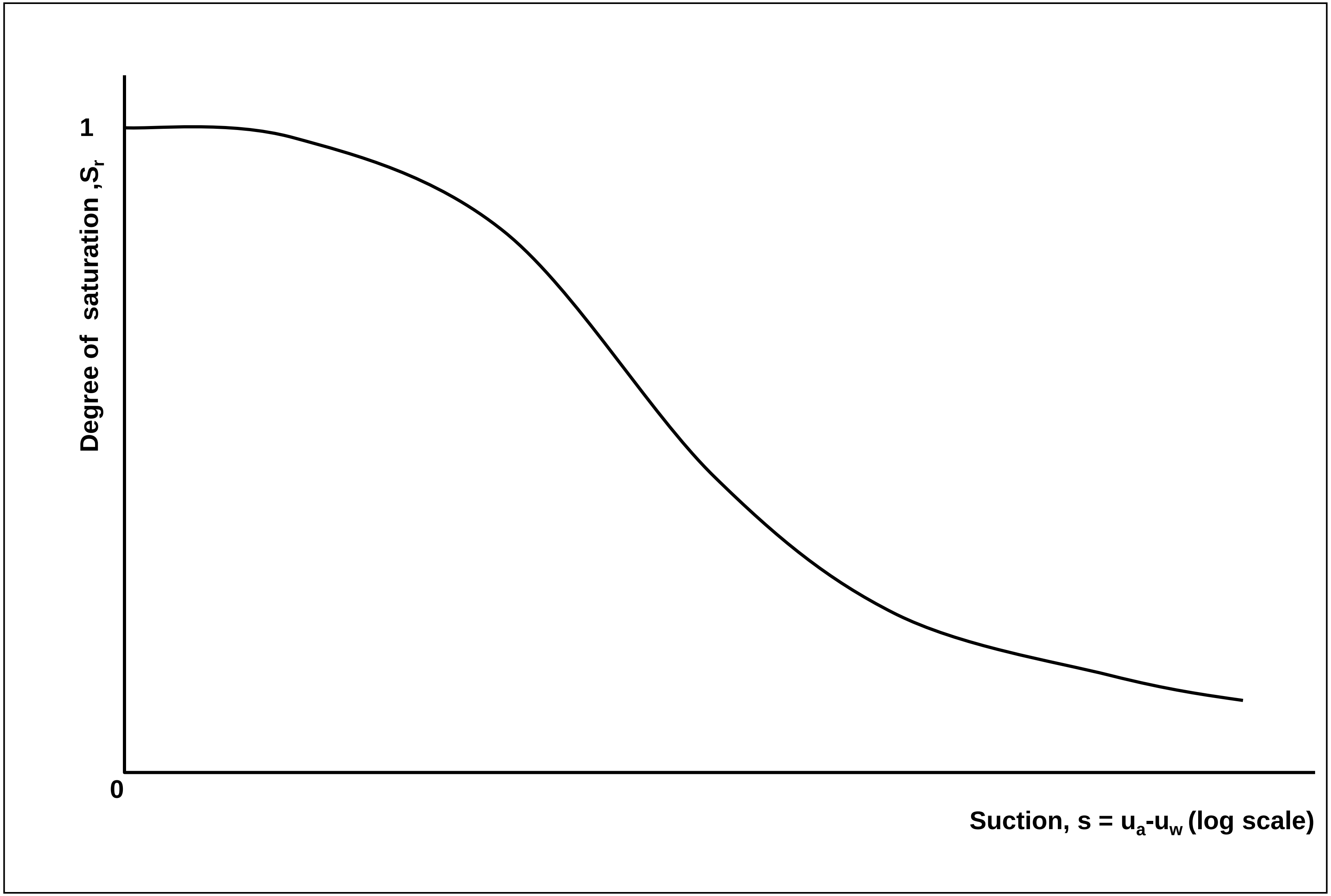
T is surface tension and r_1 and r_2 are the principal radii of curvature of the air-water interface

$$u_a > u_w$$

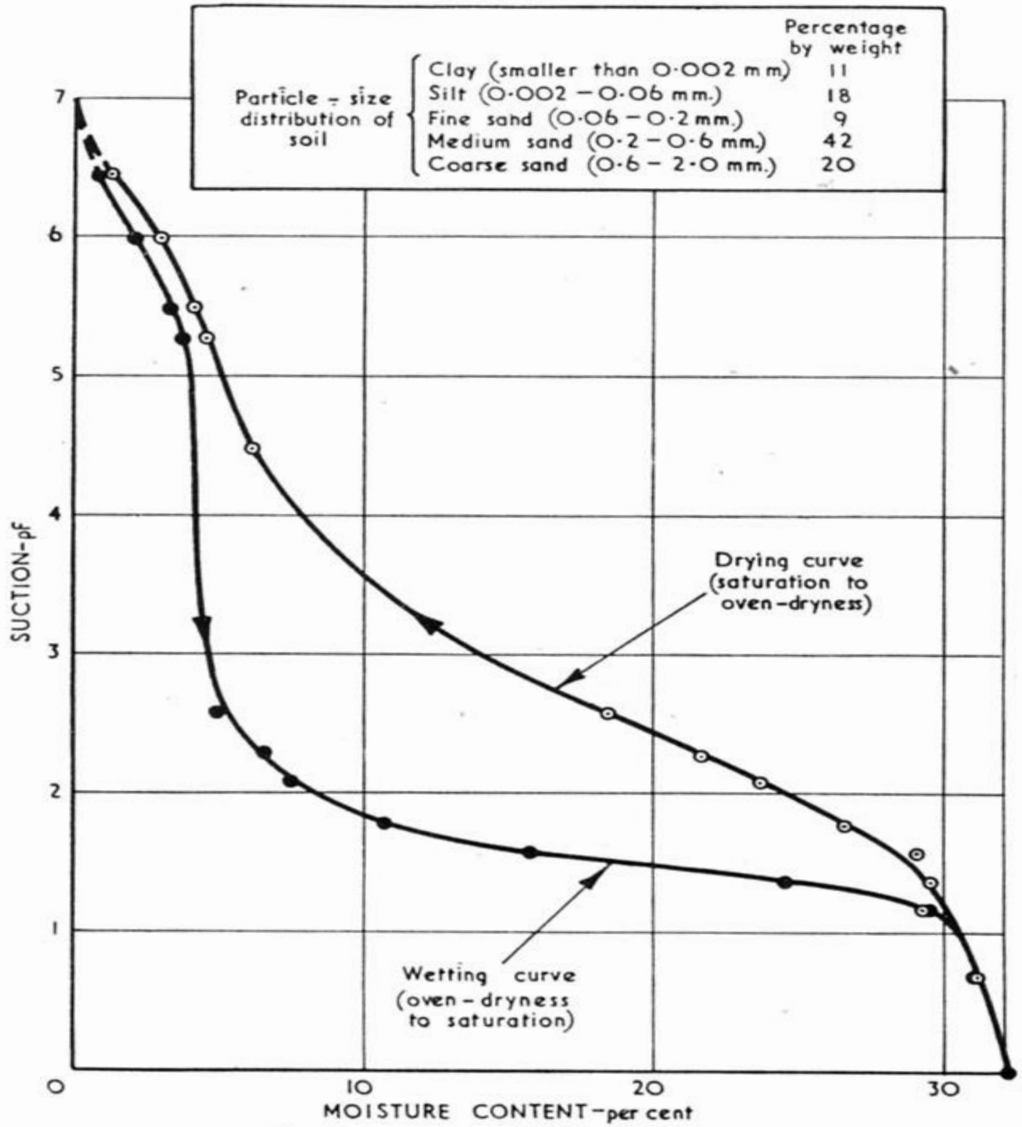
Typically: $u_a = 0$ and $u_w < 0$

$u_a - u_w$ is known as the matric suction

Water retention behaviour



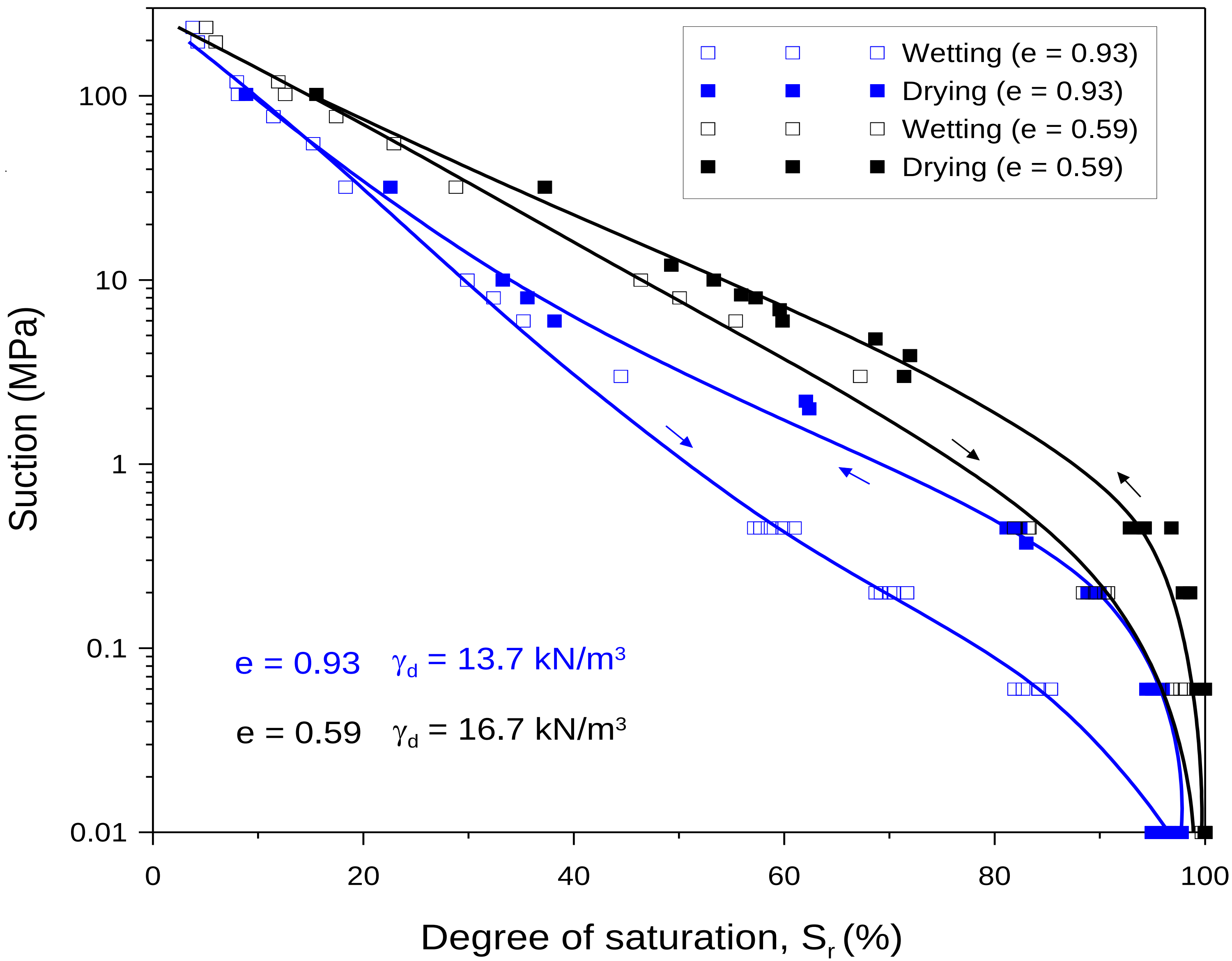
Hysteresis in retention behaviour



Relationship between suction and moisture content for a sandy soil having a small clay content

Example from Croney (1952)

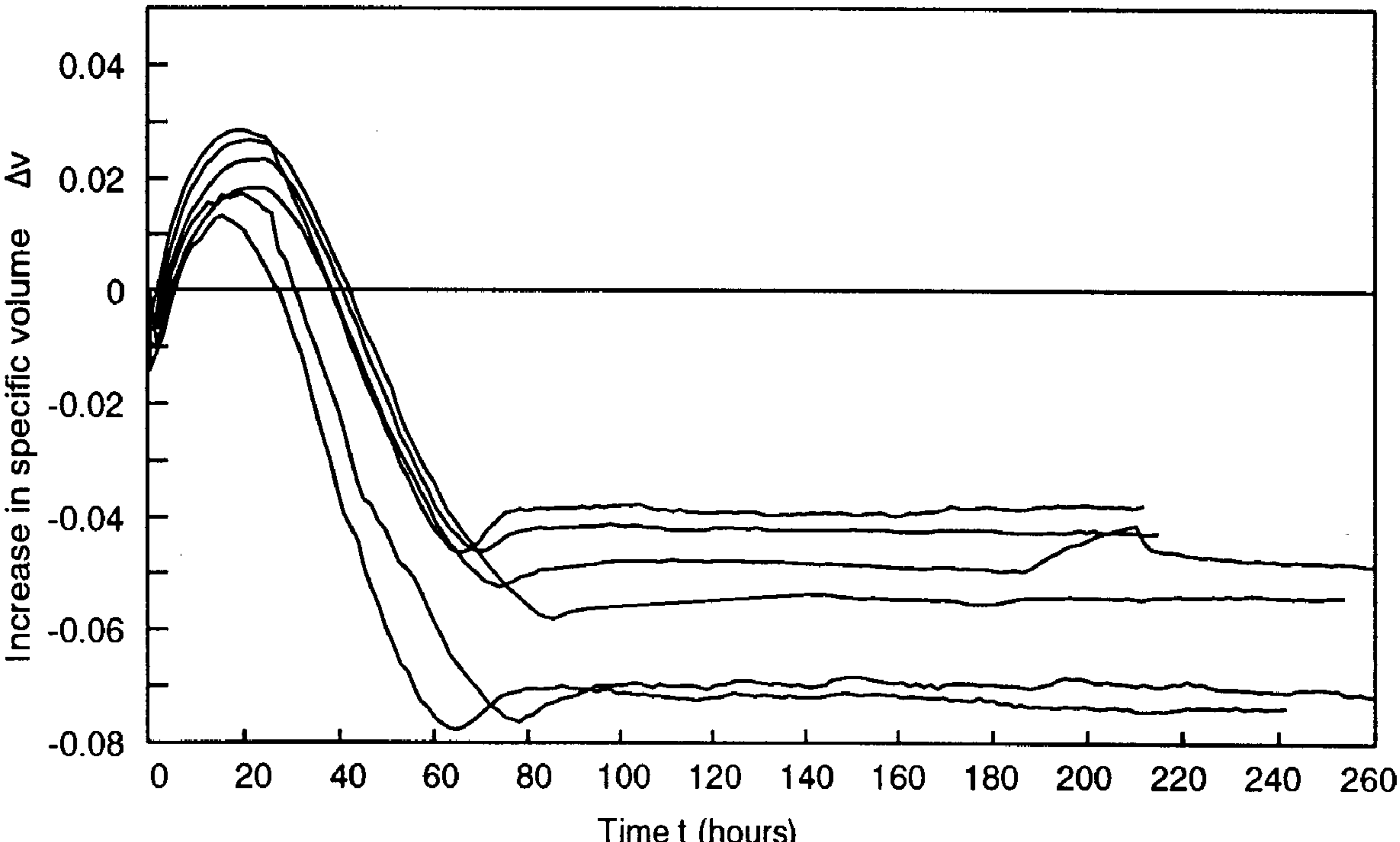
Influence of void ratio on main wetting and main drying curves for a kaolinitic-illitic soil (Romero, 1999)



Mechanical behaviour and stress state variables

- Strains caused by changes of total stresses σ_{ij} , pore air pressure u_a or pore water pressure u_w.
- Includes volumetric strains. Volumetric strains during wetting or drying are a particular issue:,.
  - Wetting: produces either swelling or collapse compression (of a single soil i.e. classification into expansive or collapsible soils can be unhelpful)
  - Drying produces shrinkage
- Includes shear strength variation with σ, u_a and u_w
- A mechanical constitutive model describes the full stress-strain behaviour.

Wetting of compacted kaolin (Sivakumar, 1993)



Single effective stress?

Saturated soils:

$$\sigma' = \sigma - u_w$$

"All the measurable effects of a change of stress, such as compression, distortion and a change of shearing resistance are exclusively due to changes in effective stress" (Terzaghi, 1936)

Unsaturated soils:

Bishop (1959) proposed:

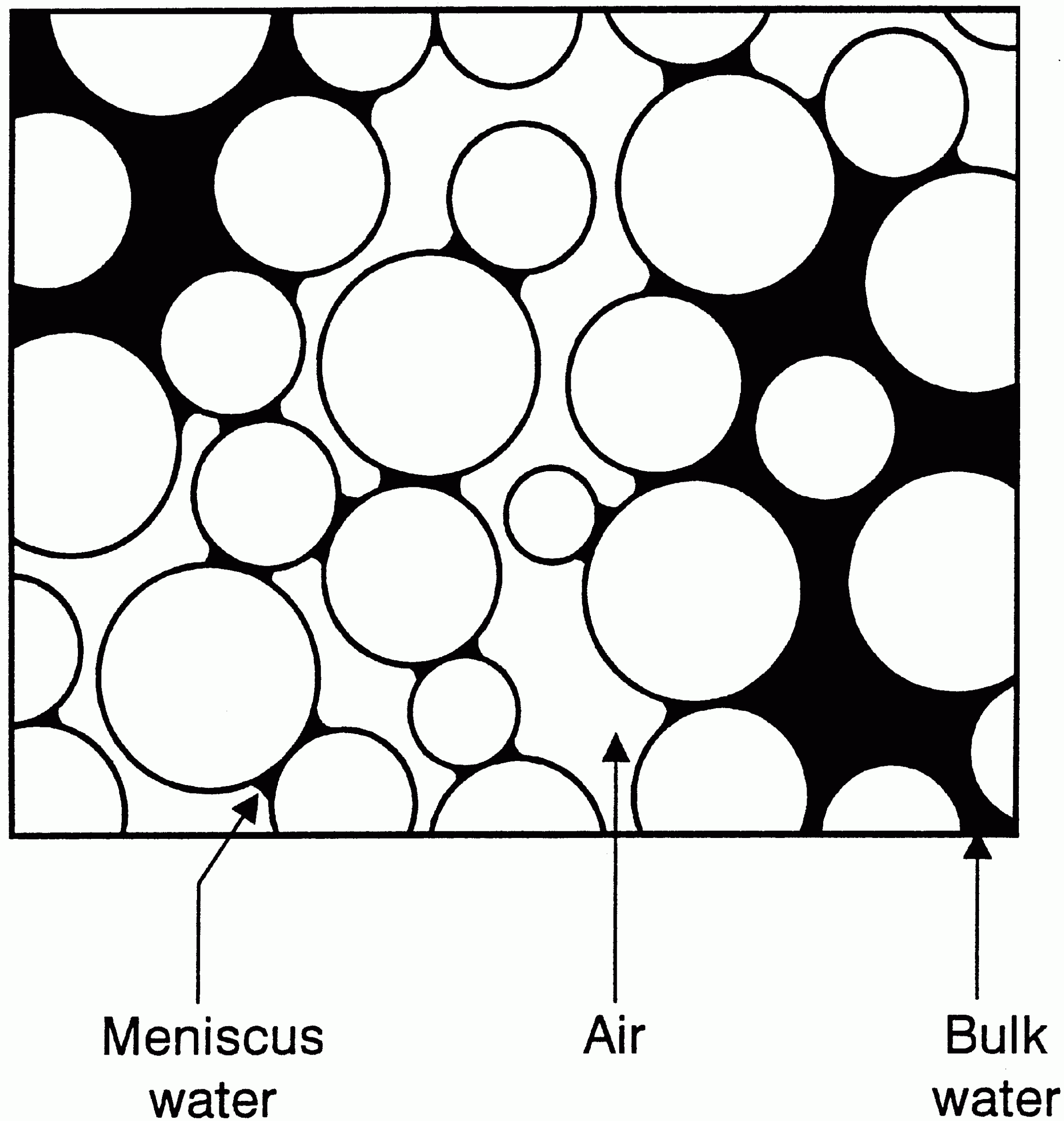
$$\sigma' = \sigma - \chi u_w - (1-\chi)u_a$$

where $\chi = f(S_r)$, with $\chi = 1$ for $S_r = 1$ and $\chi = 0$ for $S_r = 0$

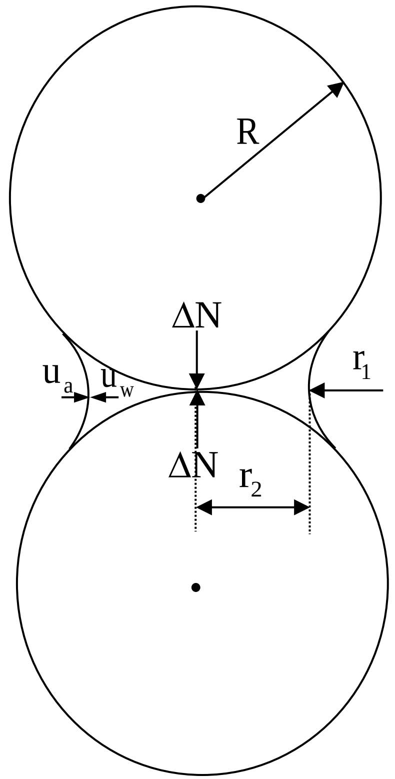
- Validity first challenged by Jennings and Burland (1962)
- Can represent shear strength variation
- Cannot represent volume changes (e.g. collapse compression on wetting) or full stress-strain behaviour
- It is actually the yield behaviour that cannot be explained in terms of a single effective stress

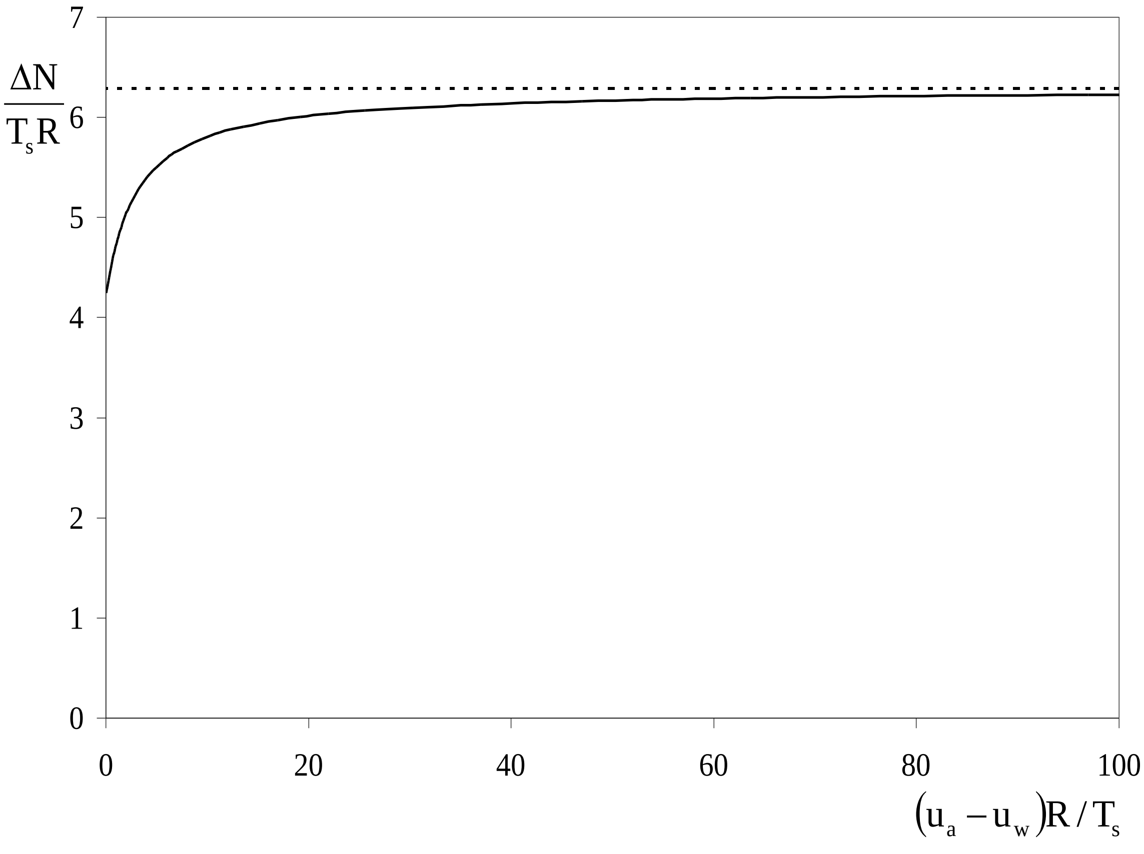
- The reason that Bishop’s single effective stress does not work is probably the influence of “meniscus water bridges” at those particle contacts that are surrounded by air-filled voids.

- A meniscus water bridge produces an additional normal component of inter-particle force at the contact, without any additional tangential component, and this creates a stabilising effect at the contact (reducing the likelihood of inter-particle slippage and hence plastic straining)


Meniscus water
Air
Bulk water

Variation with suction of additional inter-particle normal force ΔN due to a meniscus water bridge, for the case of two identical spherical particles (after Fisher (1926))





A contradiction

Wetting (i.e. a decrease of suction (increase of pore water pressure) and an increase of degree of saturation) produces:

- Elastic swelling (equivalent to a decrease of effective stress in a saturated soil) caused by the increase in pore water pressure in bulk water

- Increased likelihood of plastic strains (equivalent to an increase of effective stress in a saturated soil), as the stability of inter-particle contacts is decreased, due to the reduced number of meniscus water bridges and the reduction of suction in these meniscus water bridges

- "… no single stress variable has ever been found which, substituted for effective stress, allows for a description of *all* the aspects of the mechanical behaviour of a given soil in the unsaturated range" (Jommi, 2000)

- A second stress variable is generally required to represent the stabilizing effect of meniscus water bridges on inter-particle contacts, and the consequent influence on yielding (e.g. the possibility of collapse compression on wetting as this stabilizing effect is reduced)

Use of two stress state variables

Most common choice is:

Net stress $(\sigma_{ij} - u_a)$

Matric suction $s = (u_a - u_w)$

(Coleman (1962), Bishop and Blight (1963), Matyas and Radhakrishna (1968), Fredlund and Morgenstern (1977))

For conditions of triaxial test this simplifies to:

Mean net stress $(p - u_a)$

Deviator stress q

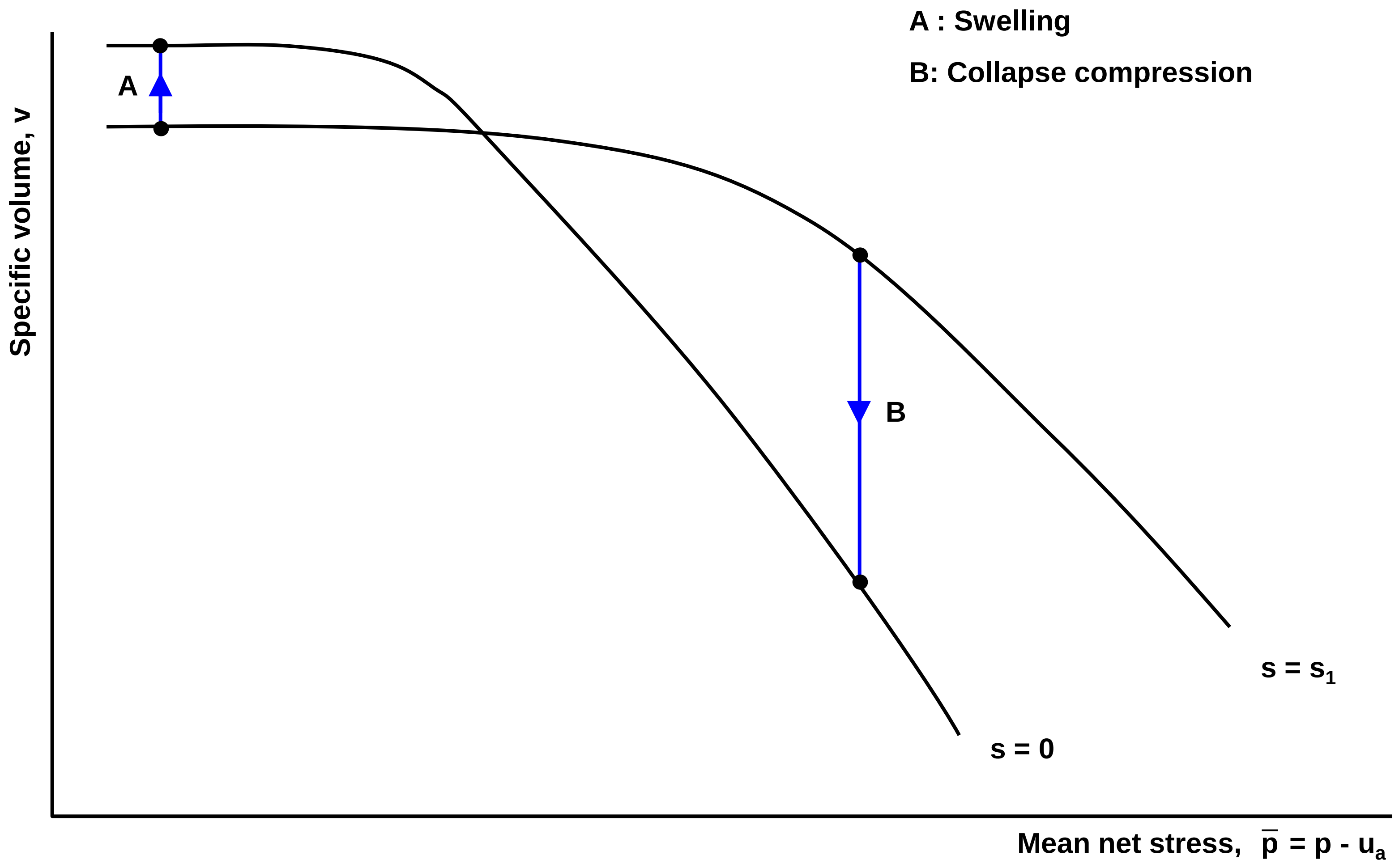
Matric suction $s = (u_a - u_w)$

Volume change

The volume change behaviour observed under isotropic loading or conditions of 1-D straining shows:

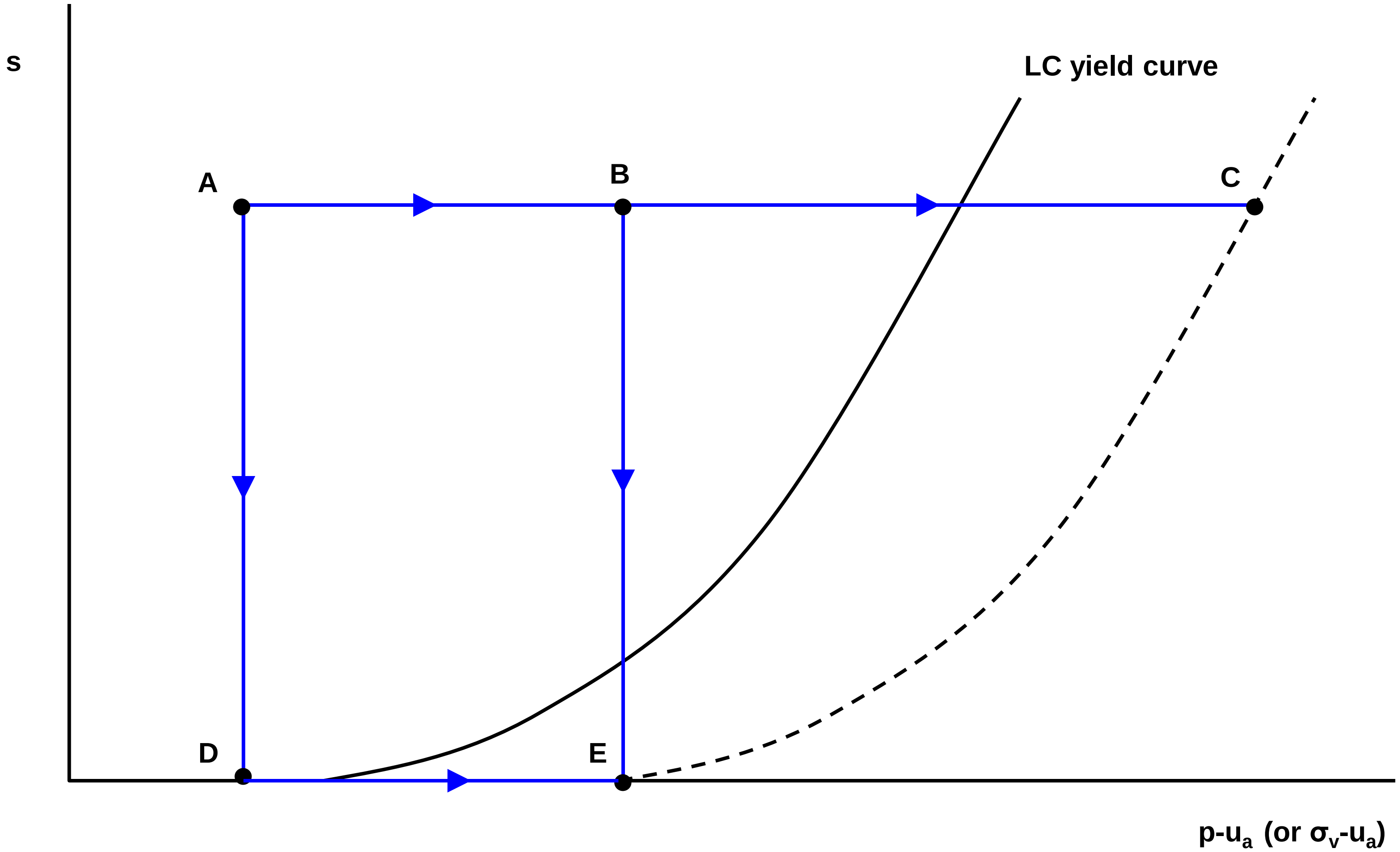
- Yield value of net stress increases with increasing *s*
- After yielding the soil follows a different normal compression line for each value of suction
- On wetting under low values of net stress the soil swells
- On wetting under high values of net stress the soil undergoes collapse compression
- On drying the soil shrinks

Isotropic loading or 1-D straining



Loading Collapse (LC) yield curve

(Alonso, Gens and Hight, 1987)



Shear strength

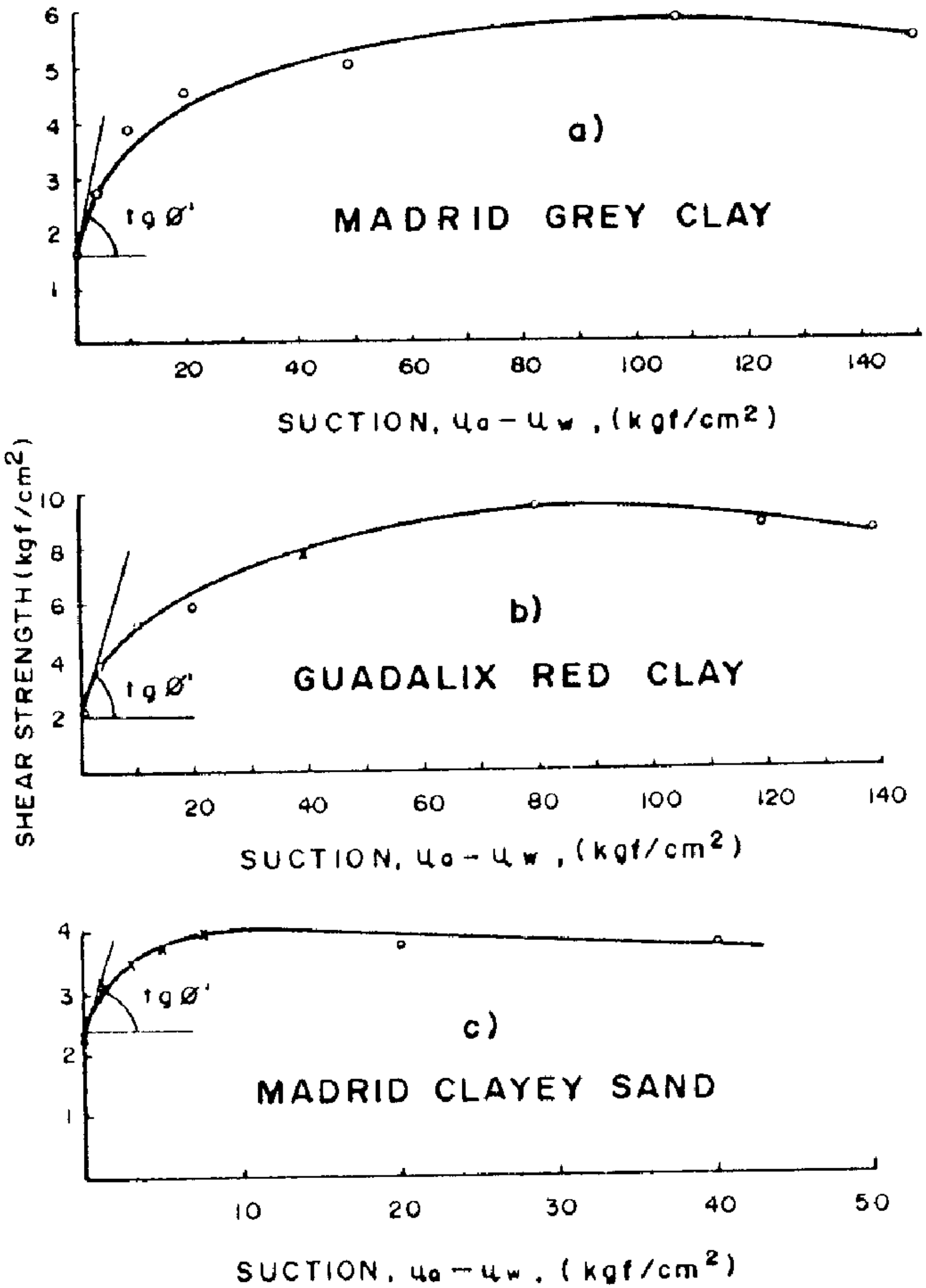
- Fredlund, Morgenstern and Widger (1978) proposed:

$$\tau_f = c' + (\sigma - u_a)\tan\varphi' + (u_a - u_w)\tan\varphi^b$$

- Subsequent work (e.g. Escario and Saez (1986), Gan, Fredlund and Rahardjo (1988)) showed that the increase of strength with suction is non-linear (φ^b not constant):

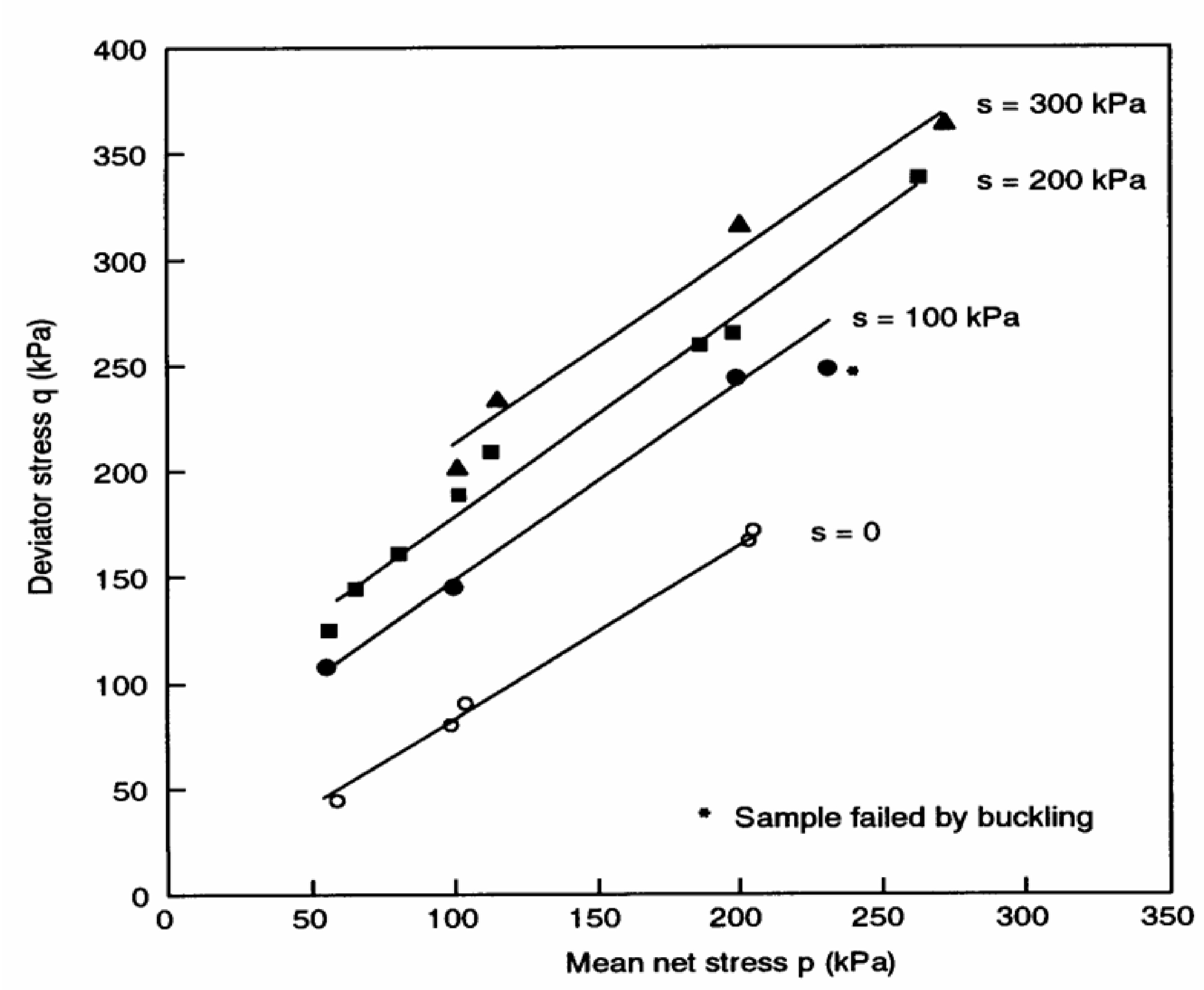
$$\tau_f = c' + (\sigma - u_a)\tan\varphi' + f(u_a - u_w)$$

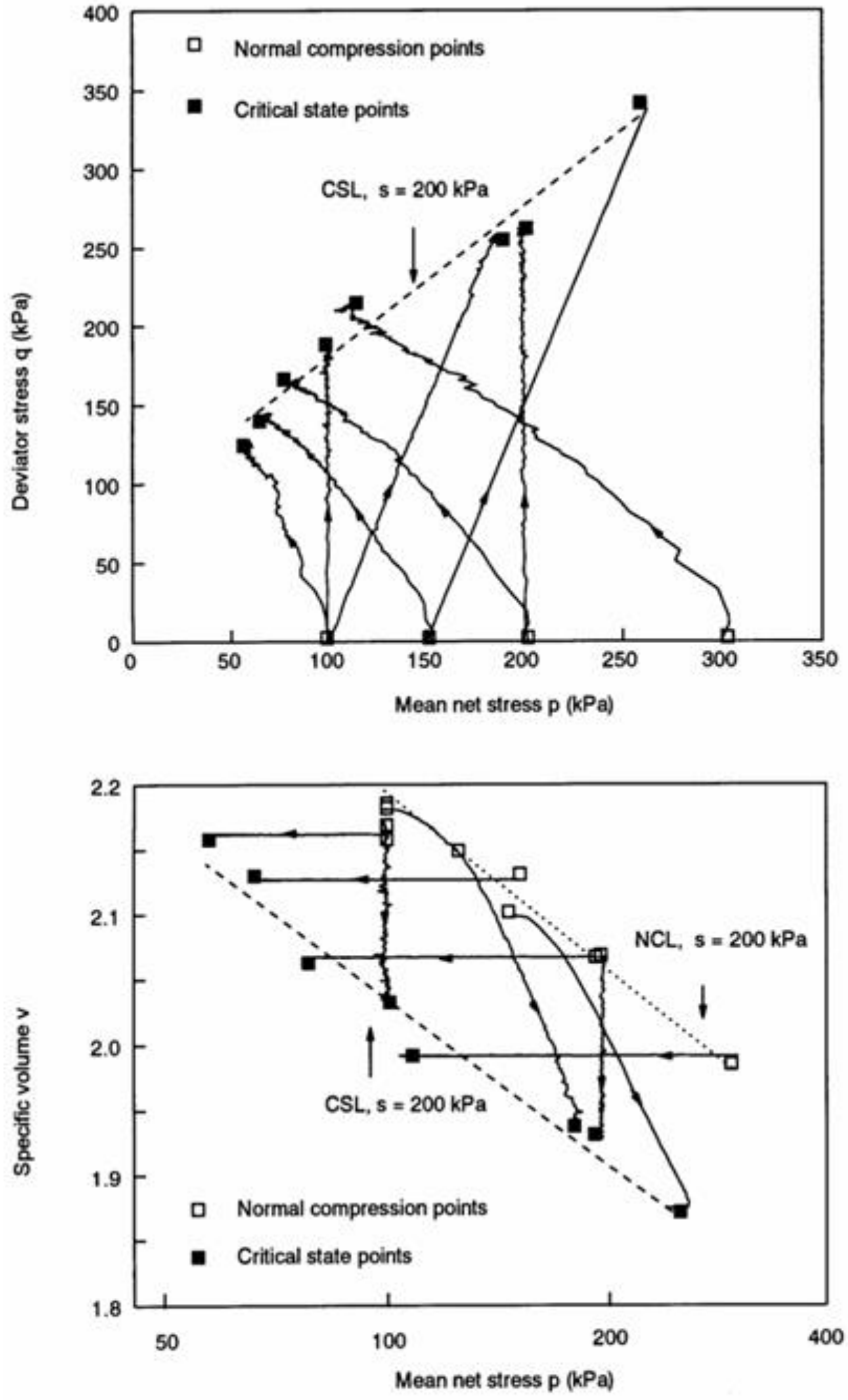
where $f(u_a - u_w)$ is a non-linear function of suction



Escario and Saez (1987)

Critical state lines for compacted kaolin (Sivakumar, 1993)

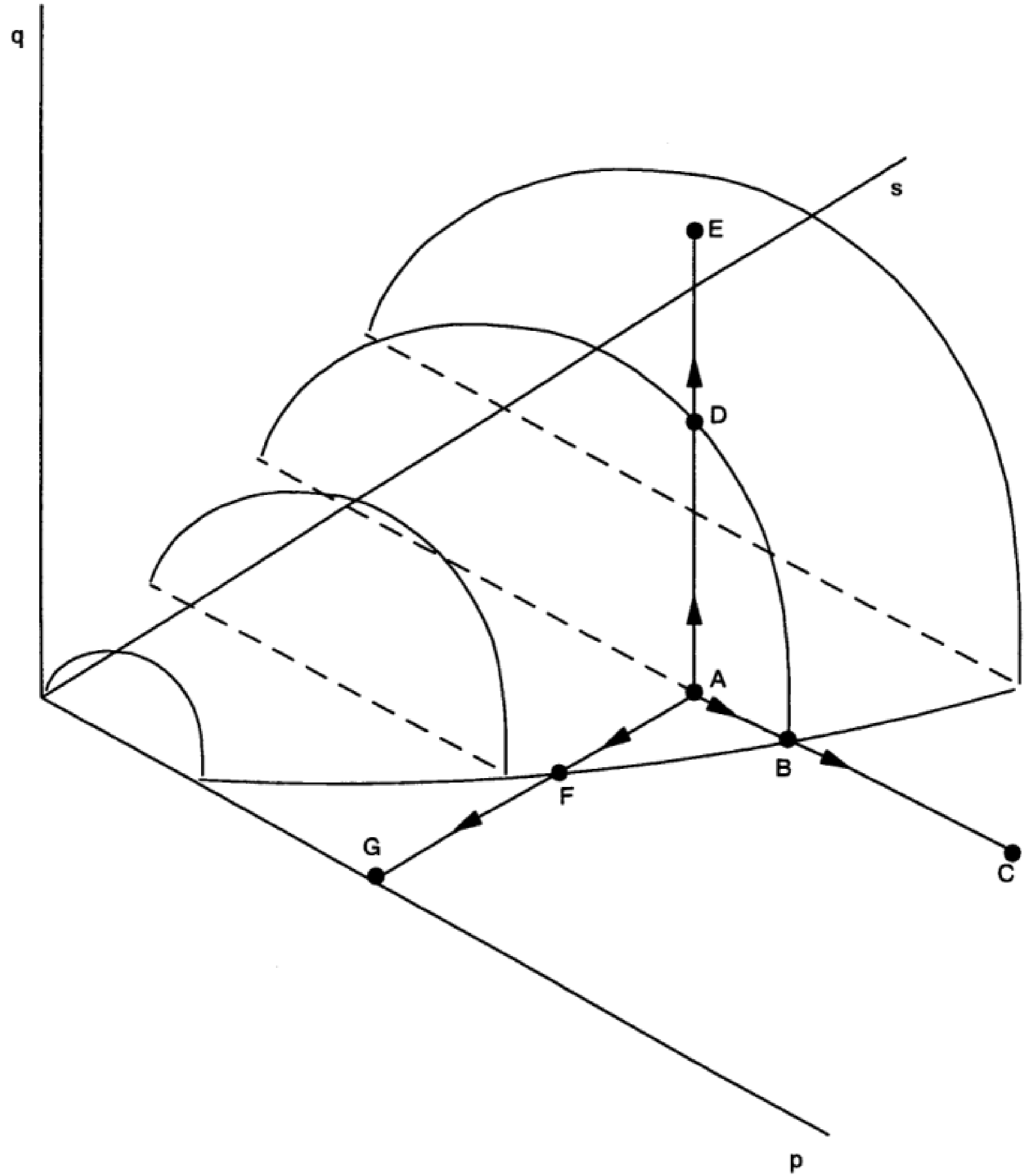




Critical state lines for compacted kaolin (Wheeler and Sivakumar, 1995)

The Barcelona Basic Model (BBM)

- An elasto-plastic constitutive framework for unsaturated soils was first proposed in qualitative form by Alonso, Gens and Hight (1987).
- First full mathematical model, known as the Barcelona Basic Model (BBM), proposed by Alonso, Gens and Josa (1990)
- The BBM is expressed in terms of net stresses and matric suction
- Tends to Modified Cam Clay in the saturated limit ($s = 0$)
- First comprehensive experimental validation of the main conceptual ideas by Wheeler and Sivakumar (1995)
- Many variants or refinements of the original BBM published since



Yield surface

BBM Equations

Elastic behaviour:

$$\mathrm{d}\varepsilon_{\mathrm{v}}^{\mathrm{e}} = \frac{\kappa d\bar{p}}{v\bar{p}} + \frac{\kappa ds}{s + p_{at}}$$

$$d\varepsilon_{s}^{e} = \frac{dq}{3G}$$

Yield surface:

$$\frac{\bar{p}_0(s)}{p^c} = \left(\frac{\bar{p}_0(0)}{p^c} \right)^{(\lambda(0)-\kappa)/(\lambda(s)-\kappa)}$$

$$\text{where} \quad \lambda(s) = \lambda(0)\left[(1-r)\exp(-\beta s) + r\right]$$

$$q^2 = M^2(\bar{p} + ks)(\bar{p}_0(s) - \bar{p})$$

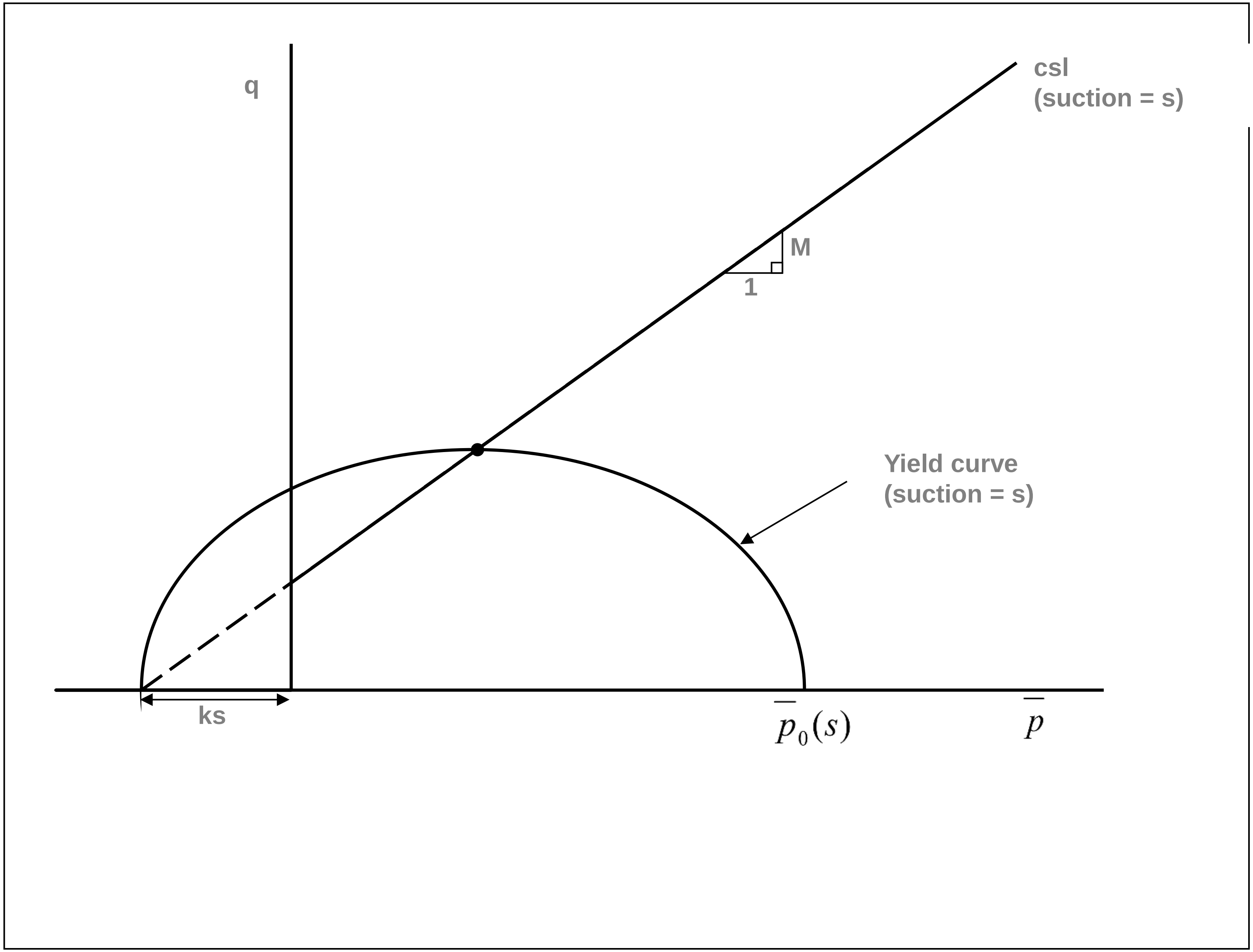
Hardening Law:

$$d\varepsilon_{v}^{p} = \frac{(\lambda(0) - \kappa)d\bar{p}_0(0)}{v\bar{p}_0(0)}$$

Flow rule:

$$\frac{d\varepsilon_{s}^{p}}{d\varepsilon_{v}^{p}} = \frac{2q\alpha}{M^2(2\bar{p} + ks - \bar{p}_0(s))}$$

Cross-section of yield surface at constant suction



BBM

Normal compression lines:

$$v = N(s) - \lambda(s)\ln\left(\frac{\bar{p}}{p^c}\right)$$

$$\text{where } N(s) = N(0) - \kappa_s \ln\left(\frac{s + p_{at}}{p_{at}}\right)$$

Critical state lines:

$$q = M\bar{p} + Mks$$

$$v = N(s) - \lambda(s)\ln\left(\frac{\bar{p}}{p^c}\right) - \left(\lambda(s) - \kappa\right)\ln\left(2 + \frac{ks}{\bar{p}}\right)$$

Achievements of BBM

- Makes sense of swelling and collapse on wetting, by appreciating that onset of collapse corresponds to yielding

- Sets volume changes during wetting/drying within a coherent overall constitutive model capable of providing quantitative predictions

- Incorporates the influence of suction on yielding under any combination of stresses

- Incorporates the influence of suction on shear strength

- Links volume changes and shearing within a single elasto-plastic model, distinguishing between reversible and irreversible strains and applicable to any stress path (equivalent to Modified Cam Clay for saturated soils)

- Most widely used elasto-plastic constitutive model for unsaturated soils

Limitations of BBM and use of alternative stress state variables

Limitations of BBM:

(a) Limitations arising from specific form of BBM

(b) Limitations arising from use of classical isotropic elasto- plasticity

(c) Limitations arising from use of net stresses and suction

(a) Limitations specific to BBM

- Cannot represent maximum collapse potential
- Often difficult to match spacing of normal compression lines at different suctions
- Arbitrary form of elastic volumetric strains
- Assumes linear increase of shear strength with suction
- Poor match of volumetric strains during shearing

Subsequent models have attempted to address each of these limitations.

(b) Limitations of BBM arising from use of classical isotropic elasto-plasticity

- Behaviour inside the “yield surface” is more complex than assumed (non-linearity at small strains, recent stress history effects, gradual onset of plastic strains, cyclic response)

- Anisotropy is ignored

- Effects of bonding and destructuring are not included

- Influence of double porosity in those soils with a bi-modal pore size distribution is ignored

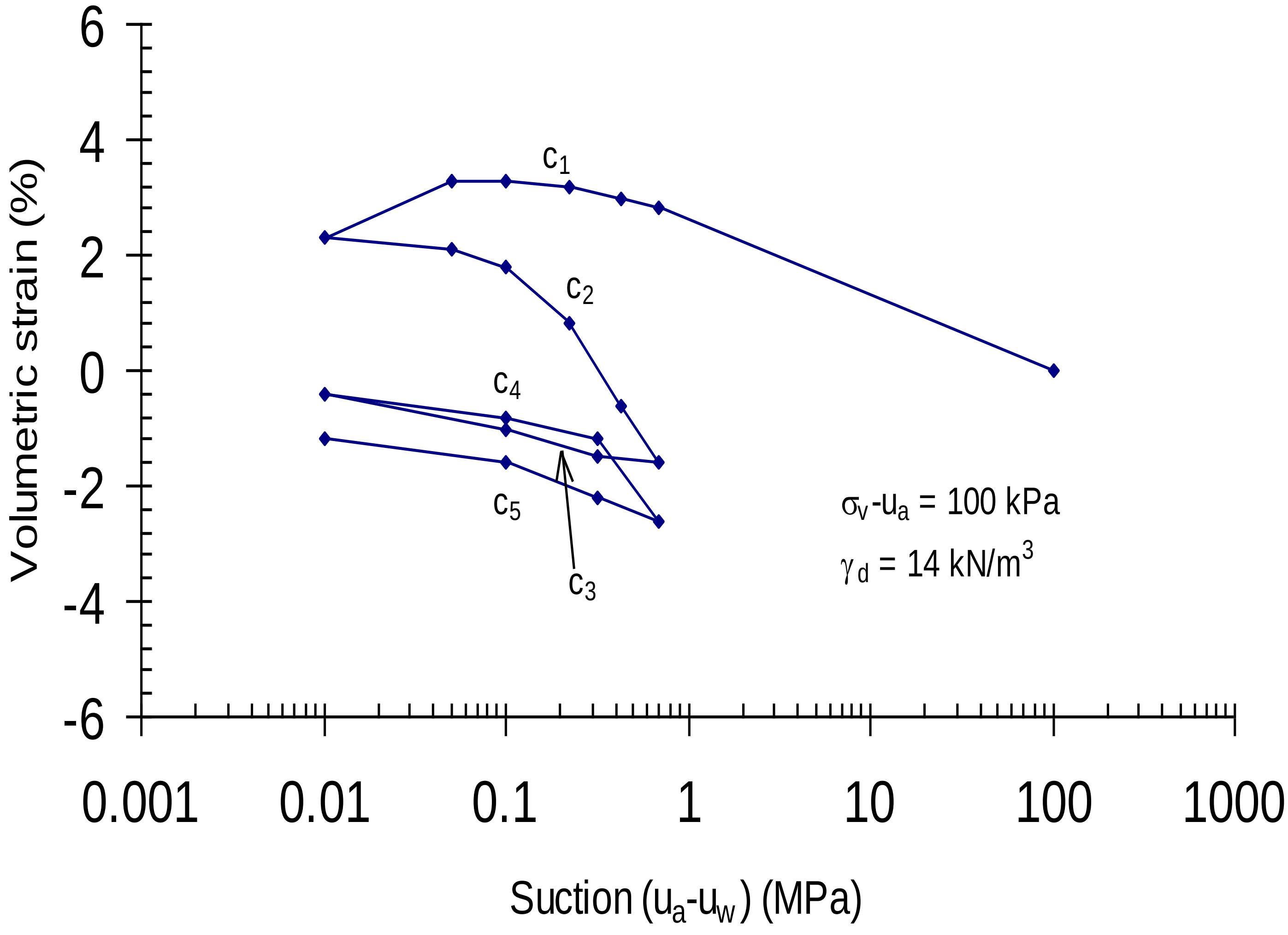
Each of these limitations has been addressed in some subsequent model.

(c) Limitations of BBM arising from use of net stresses and suction

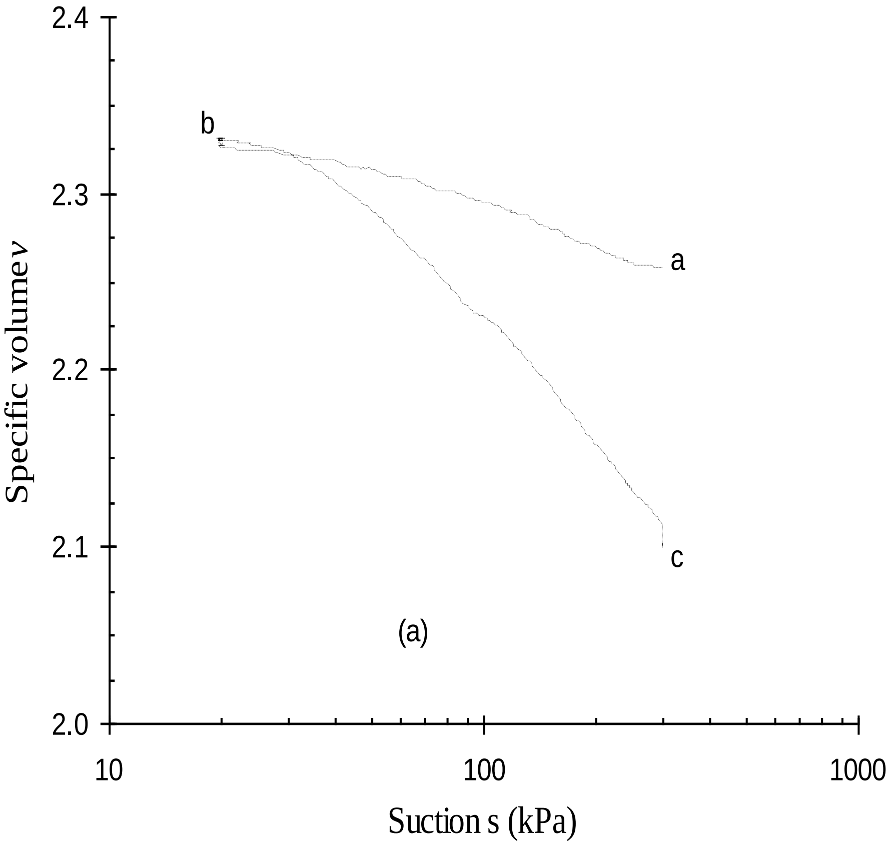
- Very difficult to incorporate the influence of S_r on mechanical behaviour and the resultant coupling between water retention behaviour and mechanical behaviour

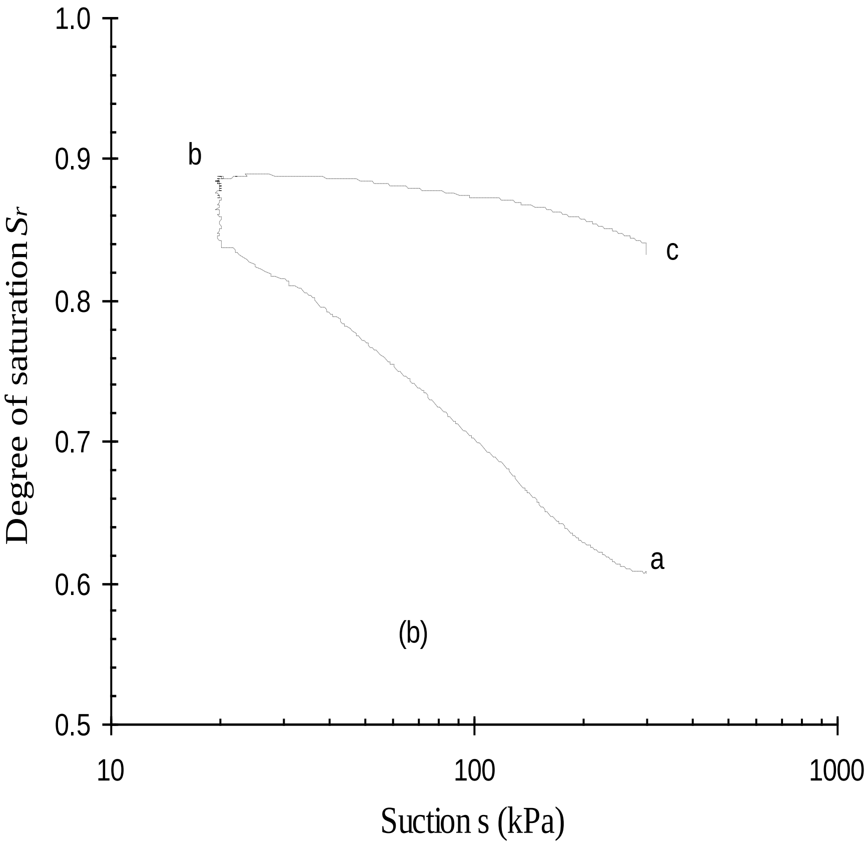
- This also means that the BBM (which converges to saturated behaviour only at $s = 0$) cannot represent well the observed transitions between saturated and unsaturated conditions, where air entry on drying occurs at $s > 0$, air exclusion on wetting occurs at a lower (but still non-zero) value of s, and both air entry and air exclusion values of s increase with decreasing void ratio.

Examples of the influence of S_r on mechanical behaviour
(a) Alonso, Lloret, Gens and Yang (1995)

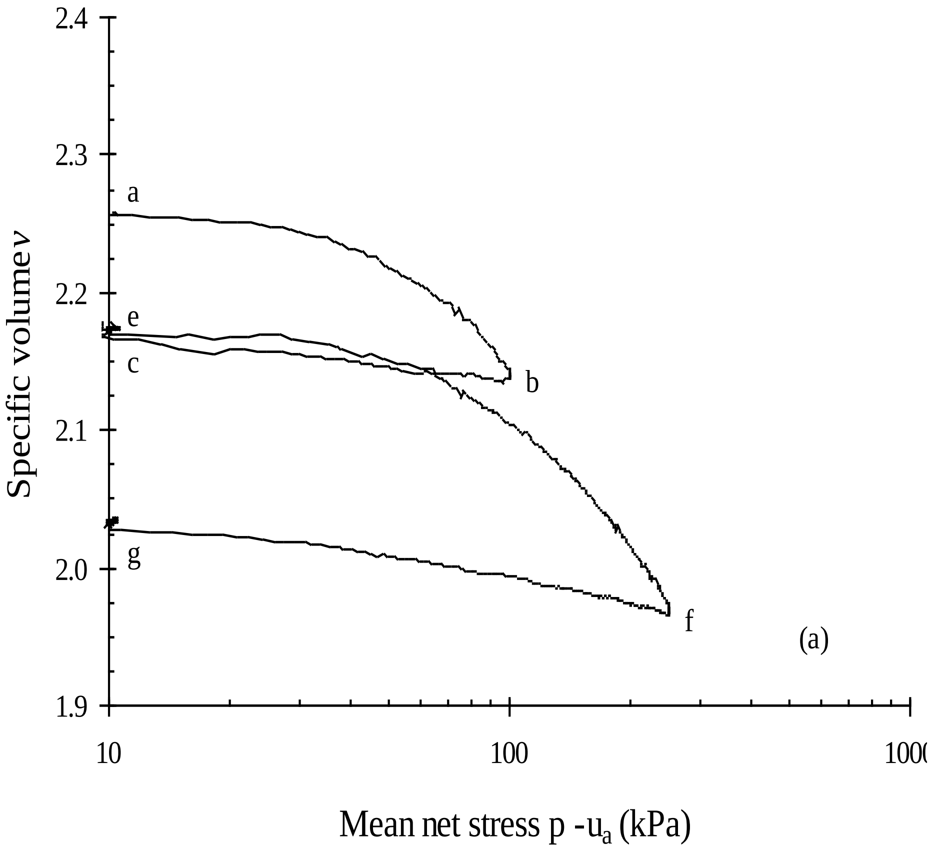


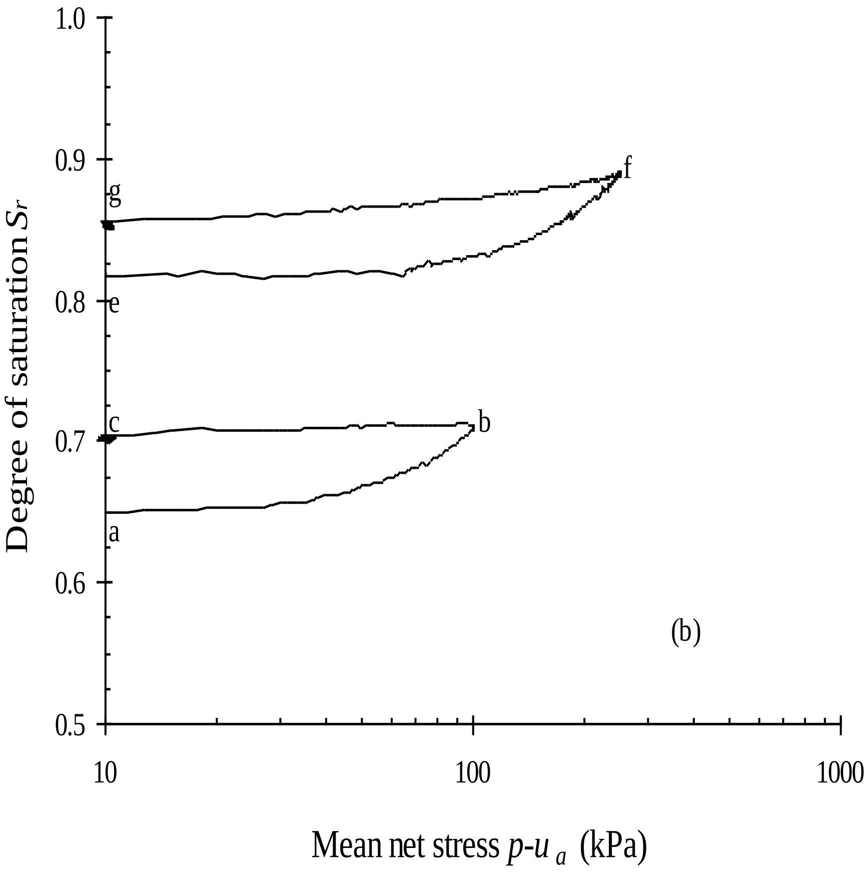
(b) Sharma (1998)

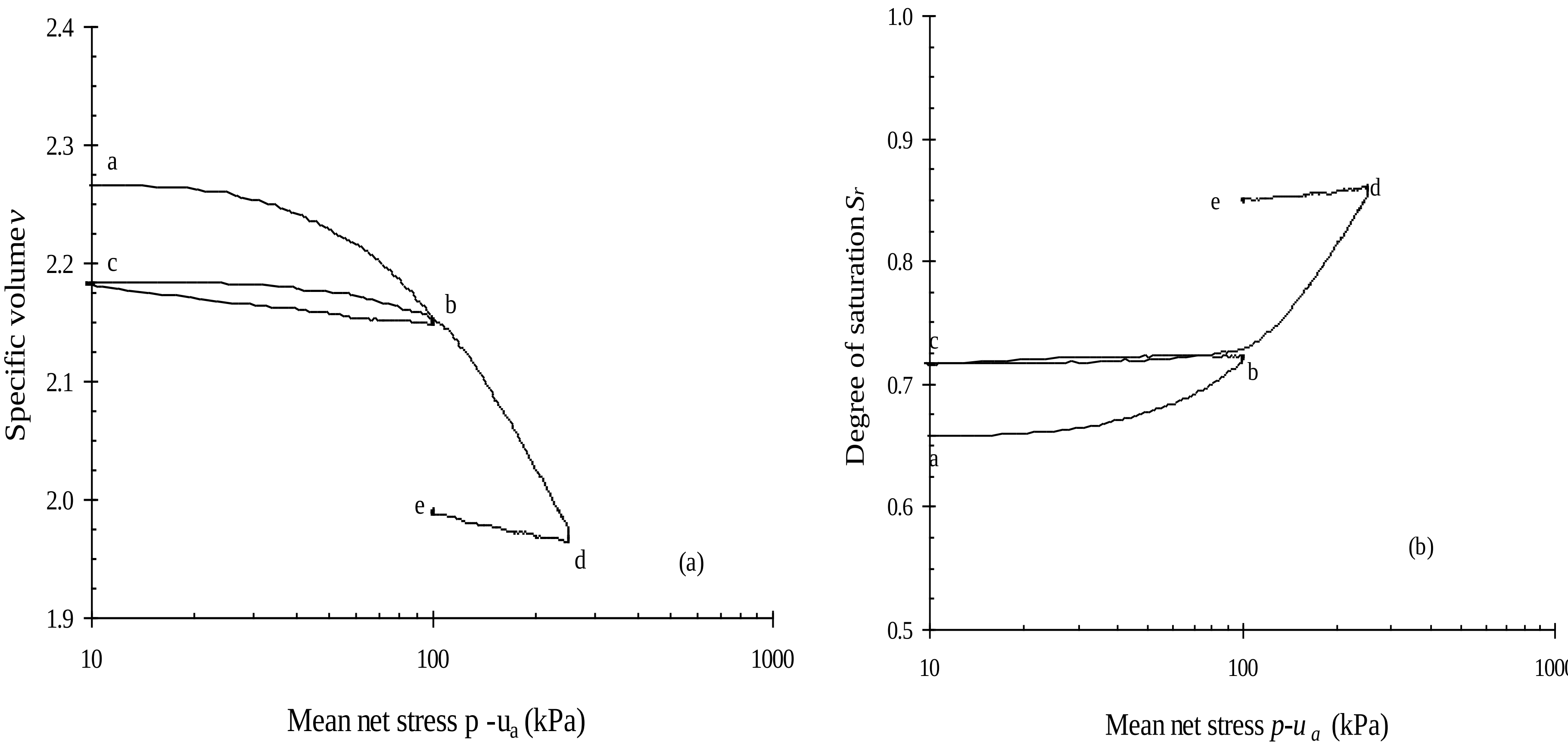




(c) Sharma (1998)







2.4
2.3
2.2
2.1
2.0
1.9
Specific volume v
a
c
b
e
d
(a)
10
100
1000
Mean net stress p -u a (kPa)
1.0
0.9
0.8
0.7
0.6
0.5
Degree of saturation Sr
e
d
c
b
a
(b)
10
100
1000
Mean net stress p-u a (kPa)

- These limitations arising from the use of net stresses and matric suction have been addressed by employing alternative stress state variables.
- Many alternative combinations of stress state variable have been proposed, generally consisting on one tensor variable and one scalar variable). See Gens, Sanchez and Sheng (2006) and Gens (2010) for reviews.
- The various combinations have been proposed based on theoretical (thermodynamic/energy) considerations, arguments about the physical phenomena involved or simple empirical observations.
- The possible advantage of some of these alternative combinations (over net stress and matric suction) is that mechanical behaviour may be represented more accurately or more easily in terms of these variables, whereas the disadvantage is that the stress variables themselves are more complex.
- In many cases the first (tensor) stress state variable is selected so that elastic behaviour and shear strength can be related solely to this stress state variable, with yield behaviour also involving (directly or indirectly) the second (scalar) stress state variable – do not be misled by those authors who refer to the first variable as the “effective stress”.

- One possible combination of stress state variables arising from both theoretical (energy) considerations and arguments about the physical phenomena is known as the “Bishop’s stress” (or “average inter-granular stress”) tensor and the “modified suction”, as used in the Glasgow Coupled Model (GCM) of Wheeler, Sharma & Buisson (2003).

- The Glasgow Coupled Model describes both mechanical behaviour and water retention behaviour in a single constitutive model, including the coupling between them (influence of plastic changes of S_r on mechanical behaviour and influence of plastic volumetric strains on main wetting and drying retention curves), whilst using few model parameters than the BBM.

Bishop's stress tensor and modified suction

- "Bishop's stress" tensor : $\sigma_{ij}^* = \sigma_{ij} - (S_r u_w + (1-S_r)u_a)\delta_{ij}$

  For mechanical behaviour, the Bishop's stress tensor accounts for the roles of total stress, pore air pressure (in air-filled voids) and pore water pressure within bulk water (water-filled voids). It does not account for the stabilising influence of meniscus water bridges.

- "Modified suction": $s^* = ns = n(u_a - u_w)$ where n is porosity

  This choice of second stress variable follows from the selection of "Bishop's stress" as the first stress variable, because the increment of work input dW to unit volume of unsaturated soil can be expressed as:

$$dW = \sigma_{ij}^* d\varepsilon_{ij} - s^* dS_r \quad \text{(Houlsby, 1997)}$$

- For triaxial tests:

  Stress variables: p^*, q , s^*

  Strain variables: $d\varepsilon_v$, $d\varepsilon_s$, $-dS_r$

Stress state variables

- Géotechnique editorial (Houlsby, 2004):

"….the unequivocal identification of the best choice of two variables on which to base the hypothesis has not, I believe, yet been achieved. It is a challenge to our readers to achieve the same breakthrough for unsaturated soils that Terzaghi did for saturated materials. We need clear definitions, empirical proof that the mechanical behaviour of unsaturated soils does indeed depend upon the chosen variables, and preferably a satisfying "explanation" in terms of well-articulated principles. It is not an easy task"

- There is probably no single correct choice! Various combinations are possible, and it may come down to convenience; considering the advantages and disadvantages of the different options

Conclusions

- Understanding of the mechanical behaviour of unsaturated soils has improved considerably over several decades
- Many aspects of mechanical behaviour can be related to variations of net stresses and suction
- The qualitative elasto-plastic framework of Alonso, Gens and Hight (1987) (i.e. the LC yield curve) is extremely helpful
- These ideas can be used for rational simplified design calculations for ultimate limit state and serviceability limit state
- Constitutive models expressed in terms of net stresses and suction (e.g. BBM) have been implemented in finite element codes and can be used for coupled hydro-mechanical analyses

Conclusions (continued)

- Some aspects of mechanical behaviour are very difficult to represent within constitutive models expressed in terms of net stresses and suction (e.g. those arising because of the influence of degree of saturation on mechanical behaviour).

- Many more recent constitutive models therefore employ alternative stress state variables.